

Generalizing Constraint Models in Constraint Acquisition

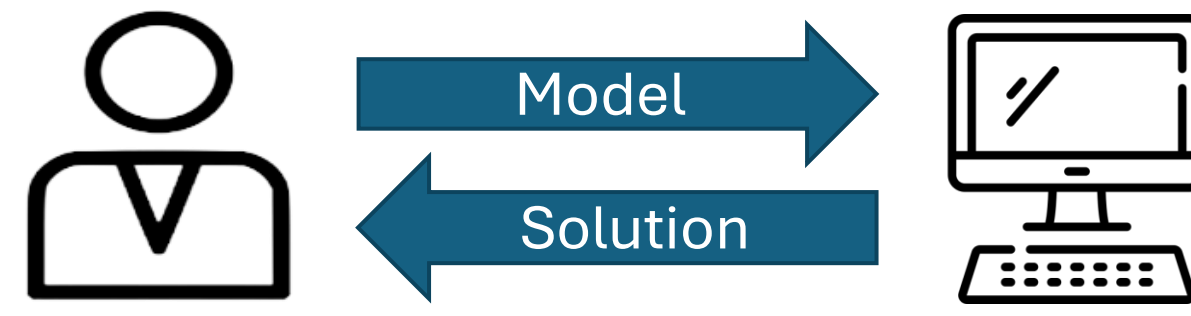
Motivation

Constraint programming (CP)

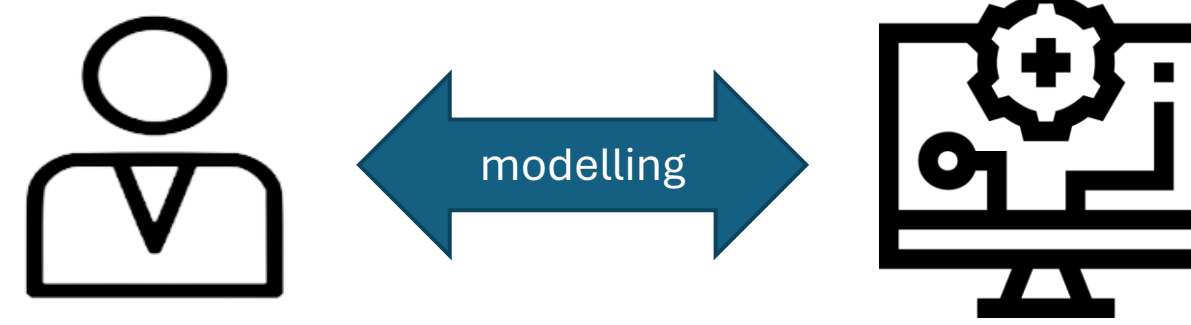
- Solve combinatorial problems

name	Week 1							Week 2							Total shifts
	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Sun	
Megan	D	D	D	D	D	D	D	D	D	D	D	D	D	D	9
Katherine	D	D	D	D	D	D	D	D	D	D	D	D	D	D	9
Robert	D	D	D	D	D	D	D	D	D	D	D	D	D	D	8
Jonathan	D	D	D	D	D	D	D	D	D	D	D	D	D	D	7
William	D	D	D	D	D	D	D	D	D	D	D	D	D	D	9
Richard	D	D	D	D	D	D	D	D	D	D	D	D	D	D	7
Kristen	D	D	D	D	D	D	D	D	D	D	D	D	D	D	8
Kevin	D	D	D	D	D	D	D	D	D	D	D	D	D	D	8
Cover D	5/5	7/7	6/6	4/4	5/5	3/5	3/5	6/6	7/7	4/4	2/2	5/5	4/6	4/4	14

- Model + Solve paradigm

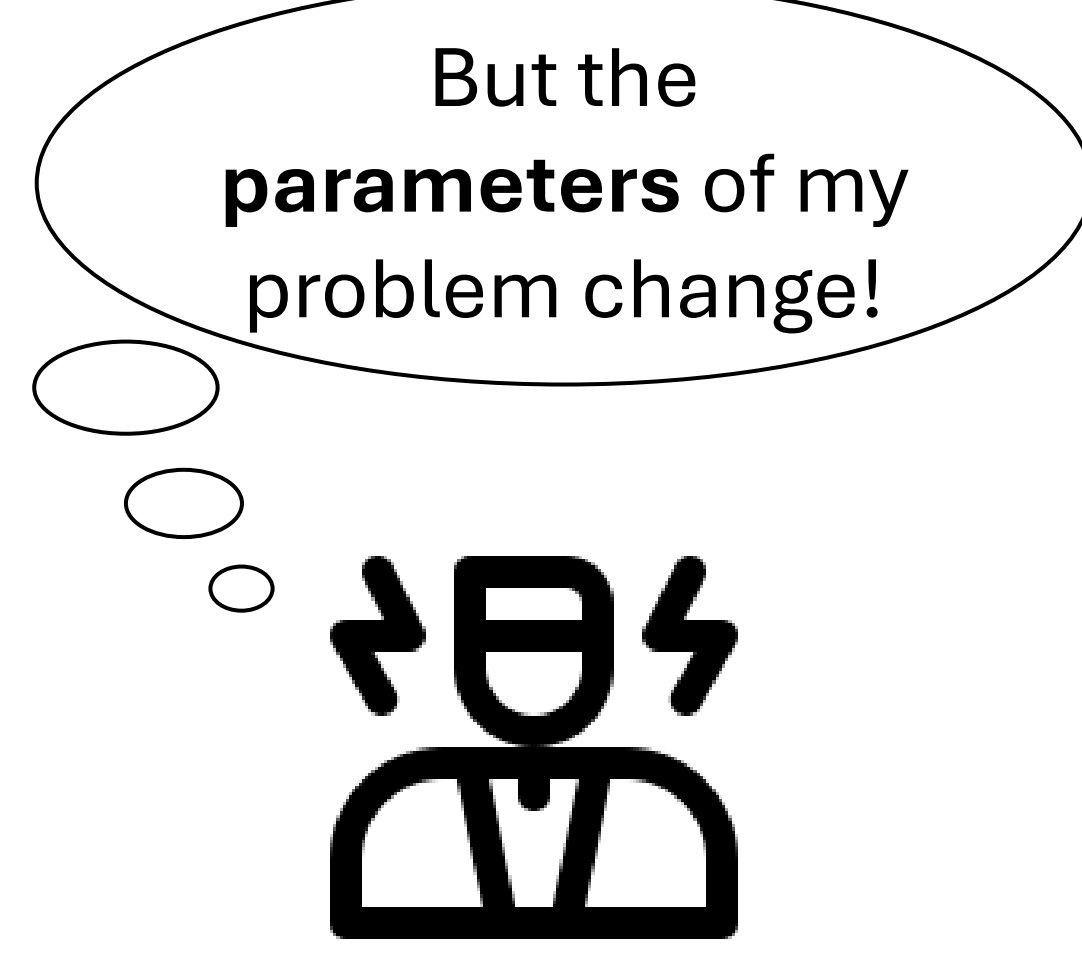
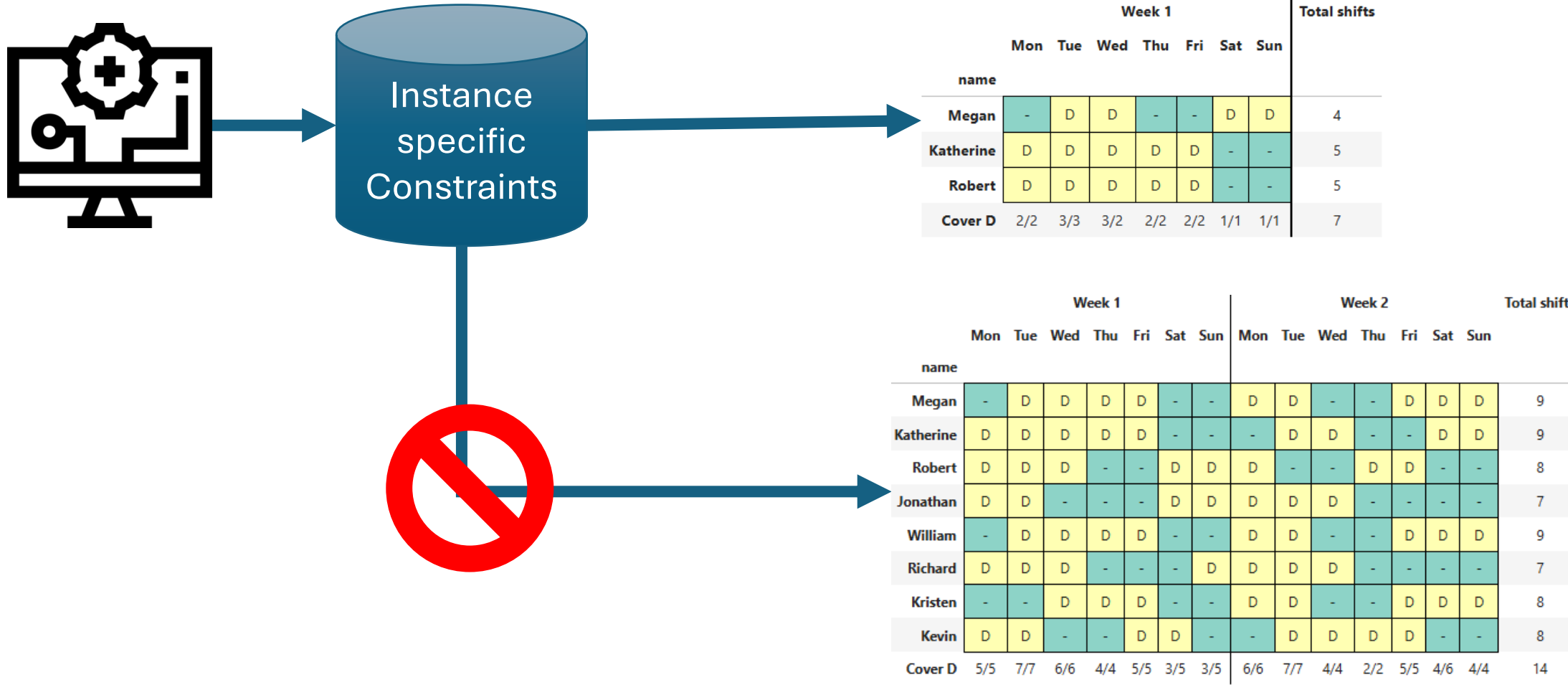


- Constraint Acquisition (CA)



Main challenge

- CA methods learn ground constraints for specific instances/parameters!
- Learned constraints cannot generalize to instances with new parameters.



Background

Parameterized Constraint Problems

An (instance-specific) CSP is a tuple (V, D, C) :

- V : Variables
- D : Their domains
- C : Set of constraints

Parameterized Constraint model

- Parameters P (e.g. number of nurses, days etc)
- Set of **Constraint specifications**
 - That can generate the constraints for any instance

Constraint specification (CS):

A triple (r, G, S) :

- The **relation** r
- Partitioning** function: $G: V_T \rightarrow P(V_T)$
- Sequence **conditions** S

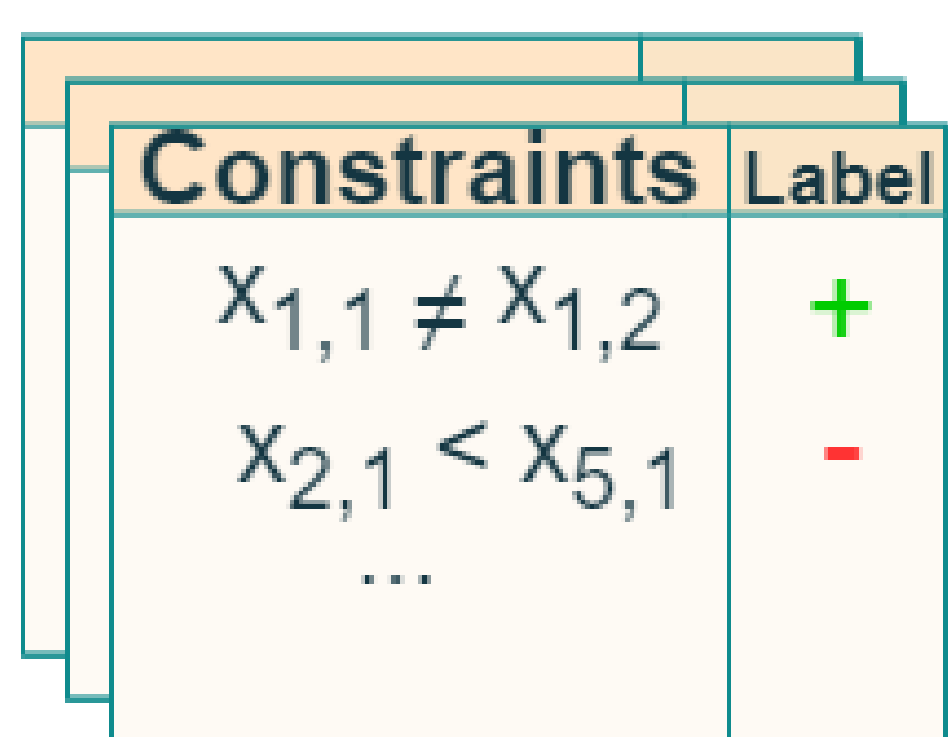
Foreach $Y \in P(V_T)$:
 Foreach scope \in combinations(Y , arity(r), S):
 $c \leftarrow (\text{rel}(c) = r, \text{var}(c) = \text{scope})$

Feature Representation of Constraints

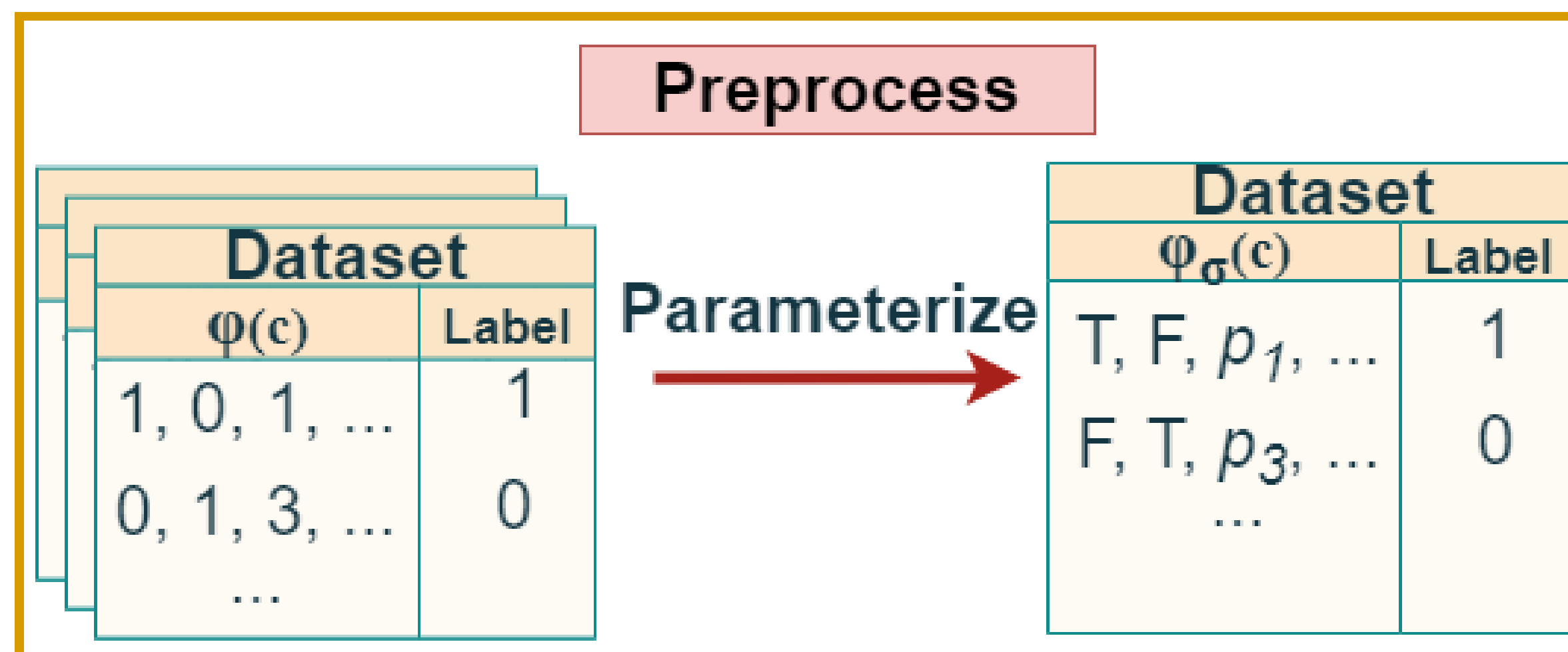
Features to capture elements of CSs

- Relations
- Partitioning functions
- Sequence Conditions

Create dataset of 'true/false' constraints



Featurize



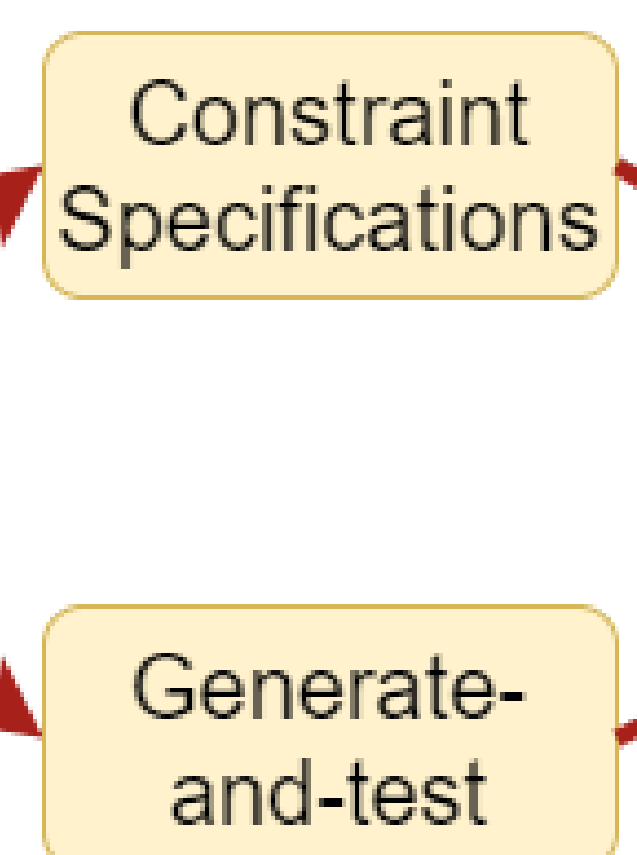
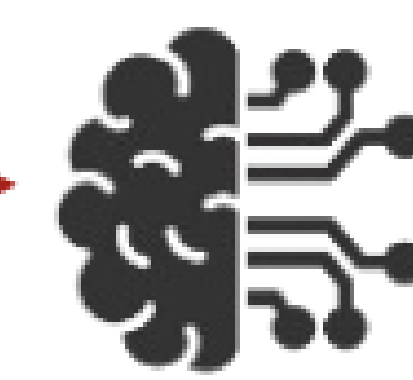
Contributions

Discretise numerical features based on the parameters

Exploiting the power of statistical ML to generalize a ground CSP to different problem instances

Generalize

Train Classifier



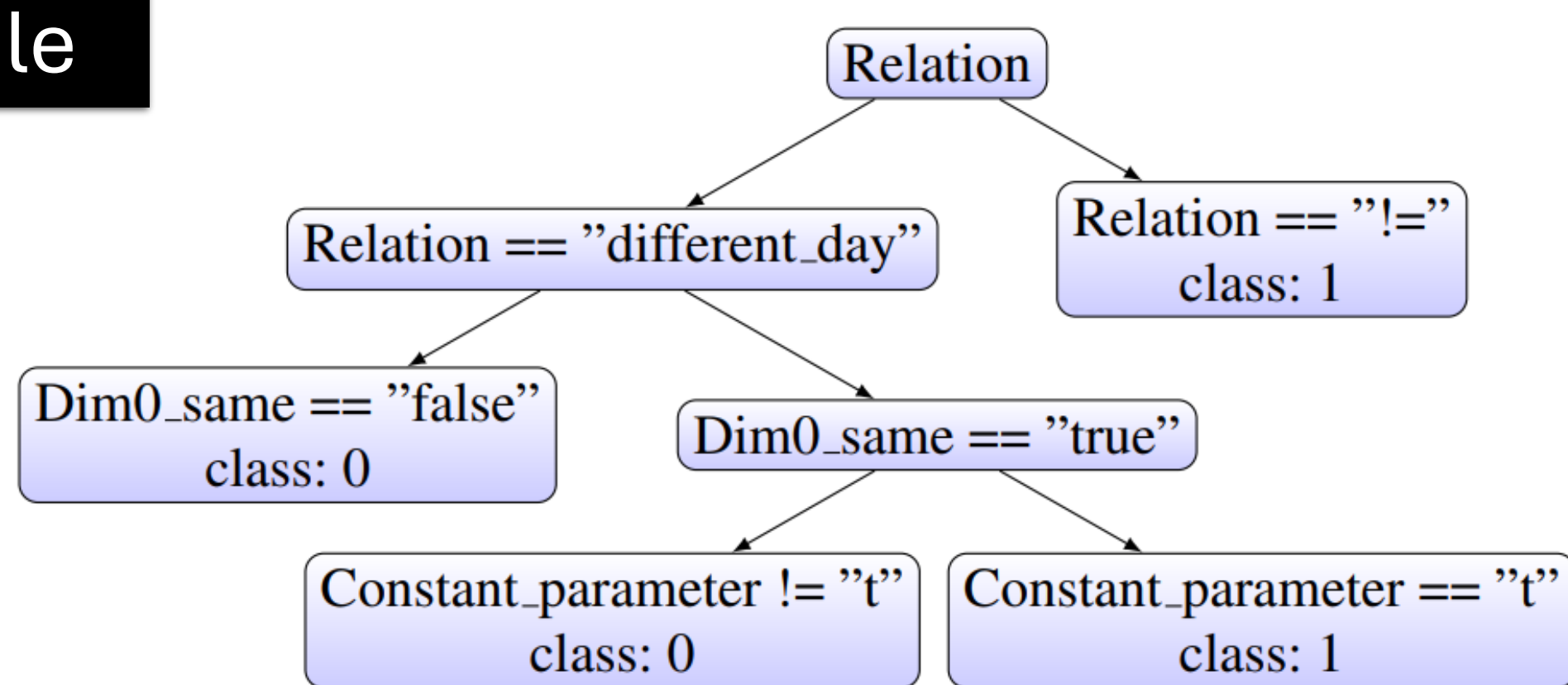
Use of Statistical ML to capture the problem structure implicitly.
 Extract interpretable Constraint specifications from learned decision rules

Extracting Constraint Specifications - Example

Exam Timetabling:

- "all courses must be scheduled in different timeslots"
- "Same semester courses must be scheduled on different days"

Decision Tree learned



Decision Rules

```

r1: Relation == "different_day"
    & Dim0_same == "false"
    then 0
r2: Relation == "different_day"
    & Dim0_same == "true"
    & Constant_parameter != "t"
    then 0
r3: Relation == "different_day"
    & Dim0_same == "true"
    & Constant_parameter == "t"
    then 1
r4: Relation == "!=" then 1
    
```

Extracted Constraint Specifications

```

Foreach row  $\in$  all_rows:
    Foreach scope  $\in$  all_pairs(row):
         $c \leftarrow (\text{rel}(c) = \text{"different\_day"}(t), \text{var}(c) = \text{scope})$ 
Foreach scope  $\in$  all_pairs(V):
     $c \leftarrow (\text{rel}(c) = \text{"!="}, \text{var}(c) = \text{scope})$ 
    
```

Evaluation

Benchmarks:

- Sudoku
- Exam Timetabling
- Golomb
- Nurse Rostering

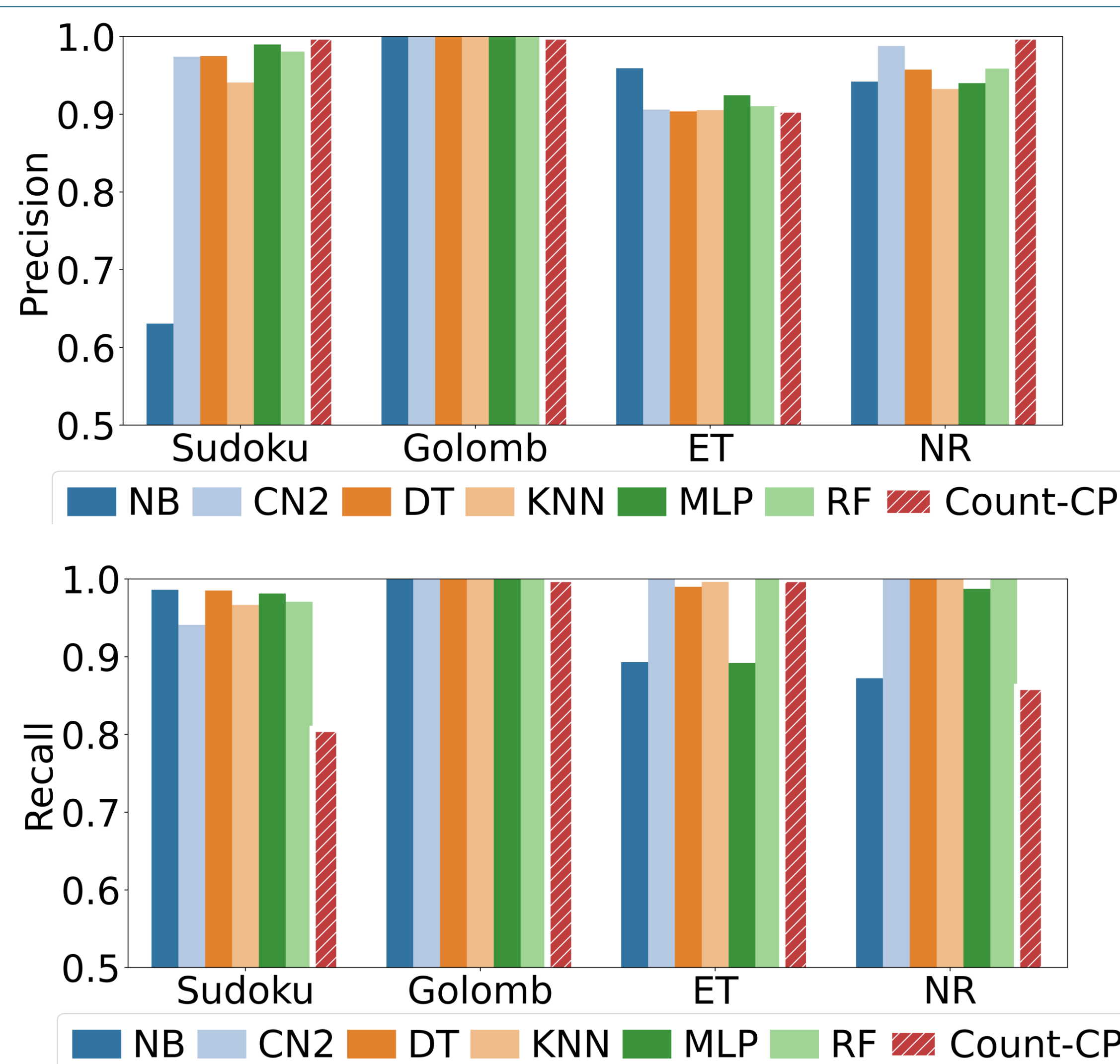
10 instances from each
 Leave-one in Cross Validation

Classifiers:

- Random Forests (RF)
- Naïve Bayes (NB)
- Multilayer Perceptron (MLP)
- Support Vector Machines (SVM)
- Decision Trees (DT)
- CN2 rule learner (CN2)

Comparing against:

- Count-CP Generalization



Conclusions

- Statistical ML can detect patterns in ground constraint models and effectively generalize them.
- Even when the models learned are noisy!
- Interpretable CSs can be directly extracted from decision rules.
- Generate-and-test approach can be used to generalize with non-interpretable classifiers.